Design and implementation of effective image segmentation technique using graph theory

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CHAPTER 1: INTRODUCTION

1.1 Background of the study

Graph theory is becoming a significant tool applied widely in the numerous research areas of mathematics, science, research and technology. This graph theory uses graphs were vertices or points are collected and connected to the edges which are also known as lines. In graph theory, some are directed graph where lines have a single arrow which denotes in specific direction alone. In some other graph, edges are not located which is it does not have any edges or sometimes lines may overlap but is have the same vertex alone (World.Mathigon, 2016). The combination of graph theory is previously applied in the area of mathematics alone later it has been applied to a wider range of applications. Graph theory has been applied in the research areas of biochemistry, computer science, and operations research, electrical engineering field with a specific application of genomics, computation, and algorithm, scheduling, network communication respectively. Some of the recent research areas for application of graph theory is DNA sequencing security in a computer network through the use of minimized vertex graph (Pirzada & Dharwadker, 2007). Some other applications of graph theory are minimizing traffic congestion, creating road network, evaluating the impact of environmental data and so on.

1.2 Problem statement

Over the past few decades, the image segmentation and grouping are a challenging task for computer vision (Felzenszwalb & Huttenlocher, 2004). Among different segmentation methods, graph theoretical approaches have many useful features in practical applications. It organizes the image elements into mathematically well-defined structures, making the formulation of image segmentation problem more desirable and the computation more efficient. The graph-based image segmentation is a highly effective and cost-effective way to perform image segmentation.But, it not been implemented in an efficient manner. Additionally, the previous research fails to provide versatile graph-based algorithms for a broad range of practical applications (Peng *et al*. 2013).

Furthermore, the numerous applications and a huge amount of medical image data need sophisticated software that combines high-level graphical user interfaces as well as robust and fast interactive image analysis tools. For this purpose, the graph-based image segmentation method is used. The main advantage of using this approach for formulating the segmentation on a graph and does not require discretization by the features of Combinational operators. Additionally, discusses the graph theoretical methods for image segmentation, where the problem is modeled in terms of partitioning a graph into several sub-graphs such that each of them represents a meaningful object of interest in the image. Furthermore, compare the efficiency and effectiveness of various graph-based image segmentation algorithms in medical image classification. These methods have generated significant interest in the imaging community. The study focuses on the following properties: minimal spanning trees, normalized cuts, Euler graph, and Iterated Graph cut.

1.3 Research aim

This study mainly aims to develop the framework for image segmentation by applying graph theory. The focus of segmentation is to simplify or modify the representation of an image into somewhat that is more significant and easier to analyze. Image segmentation is normally used to trace objects and boundaries (lines, dots, curves, etc.) that can occur in images.

1.4 Research objective

The main focus of this research is to applications of Graph Theory in Image Segmentation. In specific, image segmentation is to simplify the representation of a picture into something that is more genuine and simpler to understand. It is essentially used to discover the location of objects, boundaries, lines and so on in the digital images. More precisely is the process of assigning a label to every pixel in the image such that pixels with the same label share certain visual characteristics or features.

The primary objective of the this research is,

- To review of various image segmentation techniques on graph theory method which are flexible and computationally more efficient

  To develop and simplify/modify the representation of an image into more significant and easier way in order to analyze the image segmentation by applying graph theory.

  To segment the image using two different kinds of local neighborhoods in constructing the graph, and illustrate the results with both real and synthetic images.

  To analyze the quality of the image by using various segmented results. This used to analyze the efficiency and accuracy of the segmentation process.

1.5 Significance of the study

This study covers the importance of graph theory and its application towards image segmentation. The focus of this section is on segmentation methods that are based on detecting sharp, local changes in intensity. The three types of image features in which are interested as points, edges, and lines. Subsequently, graph theory is a very efficient tool in image processing which is used for filtering, segmentation, clustering and classification purposes. Thus, this theory is becoming a perfect representation of image processing and analysis.

1.6 Chapterization

The entire research is divided into Six chapters.These are, Chapter I is Introduction, described the background of the study, which includes a brief description of graph theory and its application in image segmentation. Furthermore discussed the research problem, objective, and significance of the study.Then Chapter II is Literature Review, which begins with the concept of graph theory and further explains an overview of segmentation and application of graph theory. Moreover discussed the previous studies related to various methods (Image cluster, analysis, segmentation, wavelet transform and feature extraction) and application (remote sensing, medical application, computer vision) .Chapter III is Research Methodology, which recognizes the research questions. It explains procedures and the objectives to carry out the research. A detailed description of all the research methods including sampling procedure, data collection and type of analysis to be used for the data is elaborated in this section. Chapter IV is discussed the experimental methods. Further, analyzed the performance of proposed approach using image quality analysis approach. Chapter V discussed the simulation results using Matlab software with various images like Computed Tomography (CT), Magnetic Resonance (MR), Ultrasounds (US) images. Finally, Chapter VI discussed findings and conclusion of this research.

CHAPTER II: LITERATURE REVIEW

2.1 Introduction

In real world scenario, it can be illustrated diagrammatically with a set of points joined with lines Sasireka and Kishore (2014). Normally, the points could represent computer terminals in a computer network with lines representing communication links. A mathematical abstraction of situations which focuses on the way in which the points were connected give rise to the concept called graph (Bondy & Murty, 1976; Harary, 1969). Graph theory is one of the recent research areas of modern mathematics which has witnessed a magnificent growth due to some applications in computer and communication, molecular physics and chemistry, social networks, biological sciences, computational linguistics, and in other numerous areas. In graph theory, one of the extensively researched branches is domination in the graph (Haynes *et al*., 1998). In recent years, graph theory exhibits a spectacular growth has been witnessed due to its wide range of applications in classical algebraic problems, optimization problems, combinatorial problems, computational problems, etc. This is mainly due to the rise of some new parameters that has been developed from the basic definition of domination.

2.2 History of graph theory

Among various fields of research, mathematics plays a vital role where graph theory is widely used in structural models (OK & SD, 2015). This structural arrangements of various objects or technologies lead to new inventions and modifications in the existing environment for enhancement in those fields. Graph theory exhibits significant growth in the recent mathematics for solving a complex problem which is stated by Koinsberg Bridge in 1735 (Shirinivas et al., 2010b). This problem leads to the concept of Eulerian Graph. Euler studied the problem of Koinsberg bridge and constructed a structure to solve the problem called Eulerian graph. In 1840, A.F Mobius gave the idea of a complete graph and bipartite graph and Kuratowski proved that they are planar using recreational problems. The concept of a tree, (a connected graph without cycles (Deo, 19990) was implemented by Gustav Kirchhoff in 1845, and he employed graph theoretical ideas in the calculation of currents in electrical networks or circuits. In 1852, Thomas Gutherie found the famous four color problem. In the year of 1856, Thomas. P. Kirkmanand William R.Hamilton studied cycles on polyhydra and invented the concept called Hamiltonian graph by studying trips that visited certain sites exactly once. Even though the four color problem was invented, it was solved only after a century by Kenneth Appel and Wolfgang Haken. This time is considered as the birth of Graph Theory.

Graph theory is a branch of discrete mathematics where mathematics and computer science, graph theory is the study of graphs which are mathematical structures used to model pairwise relations between objects Singh and Vandana (2014). For solving a wide range of problems, graphs are widely used because it gives an intuitive manner before presenting formal definition. To analyze the graph theory application two problem areas are considered.

1. Classical problem
2. Problems from applications

In graph theory, the basic classical problem is defined with the help of the graph theory as connectivity, cuts, paths and flows, coloring problems and theoretical aspect of graph drawing Singh and Vandana (2014). Whereas these problems from application particularly emphasis on experimental research and the implementation of the graph theory algorithms. Graph drawing (Nishizeki *et al.*, 2000) is a key topic in implementation point of view because the automatic generation of drawing graph has important applications in key computer science technologies such as database design, software engineering, circuit designing, network designing and visual interfaces.

2.3 Definition of Graph theory

Graph theory is used in the field of mathematics and computer science using graphs in the form of mathematical structures pairwise relations models between objects from a certain collection. This graph is defined as a set of objects called vertices, points or nodes connected by links called lines or edges. Through the graphical model nodes and the second set of items called edges where a graph is defined as a relationship between such sets. In this sets, each edge joins a pair of nodes where graphs are represented graphically by drawing a dot for every vertex, and drawing an arc between two vertices if they are connected by an edge through the use directed arrow drawing arrows. Every edge can give a real value which means that a graph is extended with a weight function. In the case, when a graph presents set of nodes, the weight function is a length of every mode which is stated as a weighted graph.

A graph G is a mathematical structure used to model pairwise relations between objects from a certain collection. A graph in this context refers to a nonempty set of vertices and a collection of edges that connect pairs of vertices. The set of vertices is usually denoted by V (G) and the set of edges by E(G). The edges can be directed or undirected; it depends on the example. A graph with all directed edges is called directed graph. Otherwise it is called undirected. In a proper graph, which is by default undirected, a line from point u to point v is considered to be the same thing as a line from point v to point u. In a digraph, short for a directed graph, the two directions are counted as being distinct arcs or directed edges. If a graph G is undirected, then there is no distinction between the two vertices associated with each edge. In a directed graph its edges may be directed from one vertex to another.

2.4 Application of graph theory

Patel and Patel (2013) Graphs are used to model many problems of the real word in the various fields. Graphs are extremely powerfull and yet flexible tool to model. Graph theory includes many methodologies by which this modeled problem can be solved. Authors of the paper have identified such problems, some of which are mentioned in this paper.

Shirinivas *et al*. (2010) Graph theoretical concepts are widely used to study and model various applications, in different areas. They include study of molecules, construction of bonds in chemistry and the study of atoms. Similarly, graph theory is used in sociology for example to measure actors prestige or to explore diffusion mechanisms. Graph theory is used in biology and conservation efforts where a vertex represents regions where certain species exist, and the edges represent migration path or movement between the regions. This information is important when looking at breeding patterns or tracking the spread of disease, parasites and to study the impact of migration that affects other species. Graph theoretical concepts are widely used in Operations Research. For example, the traveling salesman problem, the shortest spanning tree in a weighted graph, obtaining an optimal match of jobs and men and locating the shortest path between two vertices in a graph. It is also used in modeling transport networks, activity networks, and theory of games. The network activity is used to solve a large number of combinatorial problems.

2.6 Gaps Identified

After analyzing all the above studies, several gaps are identified. These are discussed as follows.Lürig *et al*. (1997) presented graph algorithms to find important structures and to assign multidimensional information to these structures with the help of wavelets, but they also needed to find mechanisms to determine ranges of characteristic values for interesting medical features. Pavan and Pelillo (2003) presented a method in general and it does not prove this study can apply in a variety of computer vision and pattern recognition domains such as texture segmentation, perceptual grouping, and the unsupervised organization of an image database. Liu *et al.* (2010)require a post-processing mechanism for region merging Skurikhin (n.d.)needed improvement of the edge set that can preserve semantically salient but weak edges might require combination with region-based approaches. In addition, will also need scale-space texture analysis; in particular, adding textures characteristics to the feature vectors describing polygons and their pairwise relations. Sarsoh *et al*. (2012) presented a hard clustering algorithm since its clustering results are such that the face images of each person will be in the same tree (cluster).This study faced two issues one is, in few cases face images for more than one person are lied in the same cluster and the second one is, face images of some person were partitioned into two clusters. The future study must be conducted to remove this problem.

Elmasry *et al*.(2012) implemented segmentation using only color features. For better accuracy additional features such as shape, texture is also needed to be used. This study gave only a concept of segmentation of live CT images. Peng *et al*. (2013) conducted a survey of graph theoretical methods for image segmentation. However, the study fails to provide versatile graph-based algorithms for a wide range of practical applications. Soltanpoor *et al*. (2013) used a 4-connected scheme to create image graph and does not carry the 8-connected on the r-connected scheme. Pham *et al*. (2014) method needed to improve especially for the extraction of representative pixels and the definition of vertex description vectors supporting the construction of graph. This proposed method does not carry the other types of VHR images. Mishra *et al*. (2014) the method is sensitive to unusual cases when the background in consecutive frames changes rapidly in addition to the appearance and disappearance of multiple objects in the same scene and using multiple cameras. Srinivasan *et al*. (2014)

presented an approach for segmenting retinal layers. This study does not concentrate the segmentation of retinal layers in human eyes with multiple types of pathology from different diseases such as diabetic retinopathy, macular hole, and age-related macular degeneration also needed to analyze. Yang *et al*. (2015)this study does not focus on two concerns; one is parallel processing to speed up the segmentation algorithm. The second is taking into account the texture features to compute the heterogeneity. Kale *et al*. (2015) an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm. However, it fails to define function classes for Prim's minimum spanning tree, depth-first search, Dijkstra's shortest path algorithm, and Kruskal's minimum spanning tree. In addition need to designs for the graph traits classes will be made more generic and user defined. This way the application of all the graph classes will be truly generic, and graph theory can be applied easily for image analysis.Dikholkar *et al*. (2015) analysis of different parameters used in the algorithm for generating weights regulates the execution, Connectivity Parameter, cutoff, the number of recursions. However, it fails to provide applications to segmentation in space-variant architectures, supervised or unsupervised learning, 3-dimensional segmentation, and the segmentation/clustering of other areas that can be naturally modeled with graphs.However, Due to that NCUT method using image pixel for segmentation, there are exponential numbers of possible partitions of the graph. As a result, it is computationally expensive to find the optimal partition(Zhao, 2015). Graph cut optimization: The limitation of the Graph cut optimization approach is that it requires the number of partitions to be provided by users and hence cannot fully automatically segment an image. How to automatically determine the proper partition number for different images will be studied in the future. In addition, we will also explore the possibility of incorporating the proposed recursive calculation scheme into other information entropy methods(Yin *et al*., 2014).

2.7 Summary

In this section, divided the review of literature into four section. First, briefly discussed the history and definition of graph theory. Second, discussed the application of graph theory to the various field like computer vision, chemistry, biology, remote sensing and so on. Third, discussed the previous studies related to image segmentation particularly using graph theory. Finally, identified the problem or issues of existing method.One of the advantages to formulating the segmentation on a graph is that it might not require discretization by virtue of purely combinatorial operators and thus incur no discretization errors.

CHAPTER III: RESEARCH METHODOLOGY

3.1 Introduction

Fast growing field of image processing has put many challenges for mathematicians. It includes researching the ways to improve the efficiency and accuracy in the practical applications by processing the images correctly. Digital image processing opened new avenues for interdisciplinary research for tackling such challenges and provided new directions for research. Today, it has been used for reliable personal identification and authentication techniques such as biometrical systems, medical applications, remote sensing and in many others. Image segmentation is the first stage in any attempt to analyze or interpret an image automatically which bridges the gap between low-level and high-level image processing. The graph theoretical approaches organize the image elements into mathematically sound structures and make the formulation of the problem more flexible and computationally efficient towards improving the performance of graph partitioning algorithms. The main aim of this research is to analyze the applications of Discrete Mathematics in image processing, particular the Applications of Graph Theory in Image Segmentation.

The purposes of this chapter are to,

describe the research methodology,

explain the proposed design,

describe the procedure used in designing and collecting the data,

Provide a detailed description of graph-based approach

development of efficient graph based segmentation scheme for performance improvement

3.2 Proposed method

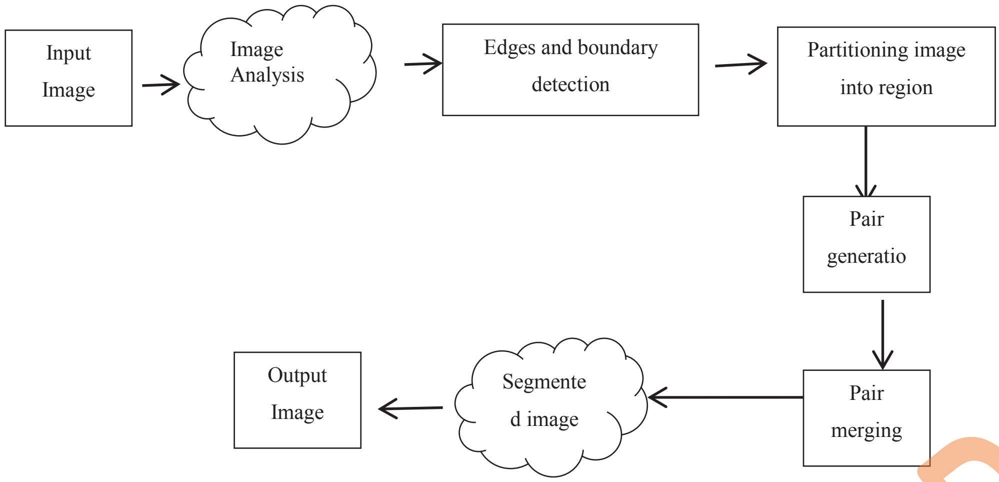
In this research proposed a novel color spatial clustering with consensus region merging for segmenting the image in an effective manner. Here the It cluster and predicate was based on measuring the dissimilarity between pixels along the boundary of two regions. By using this way, able to simplify the representation of an image into more significant and easier by applying graph theory.

This study broadly divided into three sections. Firstly, analyze and implement the existing traditional approach such as minimal spanning trees, normalized cuts, Euler graph, and Iterated Graph cut, OTSU thresholding, K-means segmentation, Split-and-merge methods, and Fuzzy Clustering method. Second, proposed novel approach and implemented using Matlab simulation software. Thirdly, compares efficiency and effectiveness of traditional graph-based image segmentation algorithms with proposed method using medical images like MRI, X-ray, and CT scan images.

3.2.1 System architecture

The system architecture consists of an image as the input, and the image undergoes the process of image analysis. Then partitioned the image and calculated the merging region. Finally, calculate the mutual information by applying the above-proposed algorithm for the images.

Figure 1: Proposed system architecture



The cluster and predicate were based on measuring the dissimilarity between pixels along the boundary of two regions. The merging predicate consists of: (a) estimating for each channel, independently of the others; (b) evaluating the eligibility of merging adjacent nodes and (c) authenticating the consistency of the merged nodes.

Let G be an undirected graph and let $v_i \quad V$-the set of all nodes corresponding to image elements, $e_i \quad E$ be the set of edges connecting to the pairs of neighboring nodes in G. Let $e_i \quad E$ be the edge connecting the vertices $v_i, v_j \quad V$, for every $i, j \quad I$ and $i, j \quad n$. Here $d(e_i) \quad w_i$ and w_i is used to measure the dissimilarity of the two nodes connected by that edge e_i.

The Predicate is defined as,

$$(P_a, P_b) \quad \begin{array}{ll} true & if\,(v_i, v_j) \quad \frac{255}{2}\log\max(n_i, n_j) \\ & \quad Eligibilit\,y(v_i, v_j) \quad true \\ & \quad Consistency(v_i, v_j) \quad true \\ False & \quad otherwise \end{array} \qquad \text{---- (1)}$$

For color RGB images, merge the regions R_i if and only if the predicate is true for each channel, independently of the others. After evaluating the merging, predicate gives

$_{i\ i}^{n} \; w_i$ 2 extremity pixels the number of times it was selected for merging.

Now let us now define the distance function between regions as

$$d_{i,j} \quad D\; f(R_i), f(R_j) \quad 0 \qquad \text{---- (2)}$$

Determine the minimum distance regions

$$(i^*, j^*) \quad \arg\min_{i,j \;\; M} d_{i,j} \qquad \text{---- (3)}$$

Where i^* and j^* are the two feature vector which is extracted from the region R. Subsequently, the minimum distance of the merge regions are,

$$R_{i*} \quad R_{i*} \quad R_{j*}$$

Remove the unused region

$$M \quad M \quad j^*$$

This recursion generates a binary tree.

Clustering can be terminated when the distance exceeds a threshold, that is,

$$d_{i^*,j^*} \quad \mathit{Threshold} \quad \text{Stop Clustering}$$

In a different number of clusters, have different threshold result. On the hand diameter or radius of the cluster. For various diameter has a different threshold value.

Let $R_m \quad S$ be the region of the image, where $m \quad M$. The term M is the simple invariant which is used to control the segmentation error.

The partitions of the image are represented by $R_m / m \quad M$ satisfies, $m \quad i, R_m \quad R_i$.

Otherwise $\;_{m \; M} R_m \quad S$

Each region R_m has features that characterize it. Any two regions may be merged into a new region.

$$R_{new} \quad R_i \quad R_j \qquad \text{---- (4)}$$

Distance between region centers is defined as

$$d_{i,j} \quad \frac{N_i}{N_{new}} \left| C_i \quad C_{new} \right|^2 \quad \frac{N_j}{N_{new}} \left| C_l \quad C_{new} \right|^2 \qquad \text{---- (5)}$$

Here $N_i \quad \left| R_i \right|$

Where N_i denotes the number of pixels of the regions R_i

C_i denotes the number of region center of R_i

$$C_i \quad \frac{1}{N_i}_{\; s \;\; Ri} \; s \text{ ---- (1)}$$

$$N_{new} \quad N_i \quad N_j \qquad \text{---- (6)}$$

$$c_{new} \quad \frac{N_i C_i \quad N_j C_j}{N_{new}} \qquad \text{---- (7)}$$

The merging order is based on a measure of similarity between adjacent regions. At each step, the algorithm looks for the pair of most similar regions (the edge of minimum cost). The similarity between two regions R_1 and R_2 is defined by the following expression:

$$O(R_1, R_2) \quad N_1 \left\| M_{R_1} \quad M_{R_1 \quad R_2} \right\|_2 \quad N_2 \left\| M_{R_2} \quad M_{R1 \quad R_2} \right\|_2 \text{ ---- (8)}$$

Where

N_1 and N_2 represents the number of pixels of the regions R_1 and R_2 respectively.

$M(R)$ - The region model

$\| . \|_2$ -the L^2 norm.

$o \; R_1, R_2$ -the order in which the regions have to be processed: the regions that are the more likely to belong to the same object. At each edge $e \quad R_1; R_2$ is associated the value $O \; R_1, R_2$.

3.2.3 Pseudo code of proposed algorithm:

// Input: Graph $G \quad (V, E)$ with number of segmentations (length)

//Output: Segmented output after cluster with region merging.

For i = 1: length

Iter=1: length % cluster

For i=no.of pairs

Check the value of predicate P with respect to its neighboring regions

Sort the edge

Each time an edge e is merged, add 1 to n

If predicate P is true,

Segmentation S is constructed.

Return S

End if

End for

End for

Remark 1: In a graph, each pair of adjacent regions is merged until the assessment of a termination criterion

Remark 2: Predicate value between two regions is true when there is exactly a cycle between them

Theorem: Let G be a graph, each pair of adjacent regions is merged until the assessment of a termination criterion. Then the segmentation S is merged.

Proof:

Let $G \quad (V, E)$ be an undirected graph, where $v_i, v_j \quad V$ is a set of nodes corresponding to image elements. It holds the following properties:

(i) $(v_i, v_j) \quad (v_i \quad 1, v_j)$: Region merge rate v_i

(ii) $(v_i, v_j) \quad (v_i, v_j \quad 1)$: Region merge rate v_j

In the context of region merging, a region is represented by a component v_{ij} V. The dissimilarity between two neighboring regions v_i, v_j V as the minimum weight edge connecting them. In a graph, there is a possibility to have at least one cycle. So, the region merging process will continue until the condition in Eq. (2) is not satisfied. Through this theorem grouping of pixels, pair will be evaluated, and it is stated that the condition required for grouping of image pixels are satisfied by the second equation. The proof of the theorem may be written in a paper and send the scanned copy for correction).

Theorem:

For the probability ratio $\quad 1 \quad O(|S| \quad)$ the segmentation S satisfying M is an over the merging of S* that is $\quad s^*(S), \ R \quad s(I): \quad R$ s(S) set of regions of ideal segmentation, s*(S) set of regions of predicted segmentation S.

Clearly state the theorem-Proof required in mathematical way. Why do we introduce this theorem here?—Not addressed.

Proof:

Assume that S and S^* are any two regions in the image. Since the predicate value between two regions is true when there is exactly a cycle between them, the couple of regions

(v_i, v_j)-wrongly used come from segmentation S^* and whose merging satisfies $\left|v_i \quad v_j\right| \quad w_b(v_i,v_j)$. Here $w_b(v_i,v_j) \quad \sqrt{w_b{}^2(v_i)+w_b{}^2(v_j)}$ and the predicted merging is $p(v_i,v_j)$. Using the fact that m holds together with this property, first rebuild all true regions of S, and then eventually make some more merges: The segmentation attained over the merging of S with high probability, as claimed.

3.3 Research design and approach

This section discusses the representative methods of graph-based image segmentation. Image segmentation is the technique of allocating a label to each pixel in an image such that pixels with the same label share some image characteristics(Foreground, the background of image pixel). The result of image segmentation is a set of segments or regions that together represents the entire image. Every pixel in a region is similar with respect to certain characteristic otherwise computed property, such as color, texture, intensity, etc. Neighboring regions are meaningfully different with respect to same characteristics. This study delivers many image segmentation methods based on the color, texture, edge of the image. For each class of methods, the study provides the formulation of the problem and presents an overview of how the methods are implemented. Although, the study discusses the main differences between these methods lie in how they define the desirable quality in the segmentation and how they achieve it using distinctive graph properties.

From the literature survey, it was learned that the performance of image segmentation methods is dependent on many factors such as intensity, texture, image content. Hence, a single segmentation method cannot be applied to all types of images. At the same time, all methods do not perform well for one particular image. The graph based methods achieve segmentation on the basis of local properties of the image. To segment images for applications where detailed extraction of features is necessary, consideration of global impression along local properties is inevitable. Research in scalable, high-quality graph partitioning comparable to sequential partitioning is also lagging. Efficient implementation, energy consumption and computational time are key aspects in defining the performance of these methods.

3.4 Data Collection

In this study used a Lancet database for getting medical, scientific information in the form of images. The reason behind this dataset, it's one of the largest hosts of scientific biomedical literature is indexed in almost 5,000 scientific biomedical data. From this database, various images are collected. Particularly, have used MRI, CT scan, and X-ray images. The colored MRI images are used instead of greyscale MRI images. The gathered data were tested using segmentation approach. Through the proposed approach image occurred from the patient can be analyzed with improved level of accuracy which helps the doctor for the accurate level of disease severity.

Is it possible to use images from any one of the hospitals from chennai? If so, what way our research would help the patience for better treatment?)

3.5 Data Analysis

The simulation tool used here is Matlab.The semantic features of input MRI brain image are accessed by segmenting the image and representing it as a graph model.The nodes of graphs are created from blobs, not from pixels, and the edge information is denoted by the relationship between these blobs. Then normalized cuts are used to separate the image into a meaningful pattern.

shall we cut the image (i/p and o/p) by horizontal and vertical lines to form a graph? Also try to measure the length/dots/intensity by using Cauchy's integration theorem) Then a relevance matrix (definition required—not addressed

3.6 Expected outcome

In this study, will discuss the useful methods of traditional, graph-based and the combination of both methods of image segmentation and the graph-theoretical approach to the image segmentation. Medical imaging is one of the most active research topics in image processing. The latest research in image segmentation has highlighted the prospective of graph-based approaches for various medical applications. The inspiration should be in the study of possessions of Minimal Spanning Tree, Euler graphs; shortest paths tree,Fuzzy graphs, Normalized cuts and Minimal cuts and revisit these ideas for image segmentation purposes. This study is helpful for those researchers who wish to carry out research in the field of image segmentation. From this, conclude that no general segmentation technique can be implemented for all types of images, but some techniques perform better than others for particular types of images indicating better performance can be obtained by selecting suitable algorithm or combination of suitable methods.

4.1 Introduction

Amongst the various segmentation approaches, the graph theoretic approaches in image segmentation make the formulation of the problem more flexible and the computation more resourceful. The problem is modeled in terms of partitioning a graph into several sub-graphs such that each of them represents a meaningful region in the image. The segmentation problem is then solved in a spatially discrete space by the well-organized tools from graph theory (Parihar & Thakur, 2014). The image segmentation problem can be interpreted as partitioning the image elements (pixels/voxels) into different categories. In order to construct a graph with an image, can solve the segmentation problem using graph theory. In this section discuss and analyze the importance of graph theory.

4.2 Application of Graph Theory to Image Segmentation

Graphs are very convenient tools for representing the relationships among objects, which are represented by vertices and relationships among vertices are represented by connections. In general, any mathematical object involving points and connections among them can be called a graph or a hypergraph. For examples, such applications include databases, physical networks, organic molecules, map colorings, signal-flow graphs, web graphs, tracing mazes as well as less tangible interactions occurring in social networks, ecosystems and in a flow of a computer program. The graph models can be further classified into different categories. For instance, two atoms in an organic molecule may have multiple connections between them; an electronic circuit may use a model in which each edge represents a direction, or a computer program may consist of loop structures. Therefore, for these examples, need multigraphs, directed graphs or graphs that allow loops. Thus, graphs can serve as mathematical models to solve an appropriate graph-theoretic problem, and then interpret the solution in terms of the original problem. At present, graph theory is a dynamic field in both theory and applications (Poghosyan, 2010).

Traditionally, a graph is modeled as a one-dimensional cell-complex, with open arcs for edges and points for vertices, the neighborhoods of a "vertex" being the sets containing the vertex itself and a union of corresponding "tails" of every "edge" (arc) incident with the vertex (Vella, 2005).

4.3 Graph Theoretical Approach to Image Segmentation

Image segmentation is a process of subdividing a digital image into its synthesized regions or objects which are useful for image analysis and has a wide variety of applications in security, (might be discussed in depth(scientific aproach) in applications of graph theory part in LR-may be half page or more forensic, medical and so on.Segmentation subdivides an image into its constitute objects. The level of subdivision depends on the type of problem solved. In this research examined about-about the image segmentation approach in various aspects in terms of flexibility and computational performance. Further, this research focused on analyzing the performance of graph theory on image segmentation approach. This paper will carry out an organized survey of many image segmentation techniques which are flexible, cost effective and computationally more efficient and finally discuss the application of graph theory which will be a highly efficient and cost-effective way to perform image segmentation.

Graph theory and discrete mathematics are related to each other since discrete mathematics is the branch of mathematics which studied about the discrete objects. These discrete objects are represented by the binary representation of objects like 0's and 1's. Therefore, both computer structure and operations can be described by discrete mathematics.

This makes an efficient tool for improving reasoning and problem-solving skills. Concepts from discrete mathematics are useful for describing objects and problems algorithmically and analyze the time and space complexity of computer algorithms and programming languages. In the following subsection have briefly review and discuss the various segmentation methods. Is this paragraphy required!?

4.3.1 Image segmentation

As already said, image segmentation is the process of subdividing the image or partitioning the image into its synthesized regions or objects which are useful for image analysis. Based on methods used for segmentation, it can be broadly classified into three categories; traditional segmentation, graphical theory based segmentation and a combination of both.Traditional methods approached the problem either from localization in class space using region information or from localization in position, using edge or boundary information.The segmentation method for monochrome images is mainly based on the two properties of gray level value; similarity and discontinuity.The first will partition the image based on similarity and the last based on dissimilarity.Thresholding, histogram comes under first, and split and merge come under second.Graphical methods are becoming more popular because they are providing a common framework for designing segmentation algorithms for a wide variety of applications, and also, they can be used in many prototypes. Graphical methods include Normalized cuts and image segmentation, Efficient graph-based, Iterated graph cuts for image segmentation, using minimal spanning trees and using Euler graphs.For improving the performances, both methods are used in combination. The diagram for classification of image segmentation is shown below

4.4 Image segmentation Methods

Many traditional classification methods are available for image segmentation they are discussed in the following subsections

Figure 2: Image enhancement

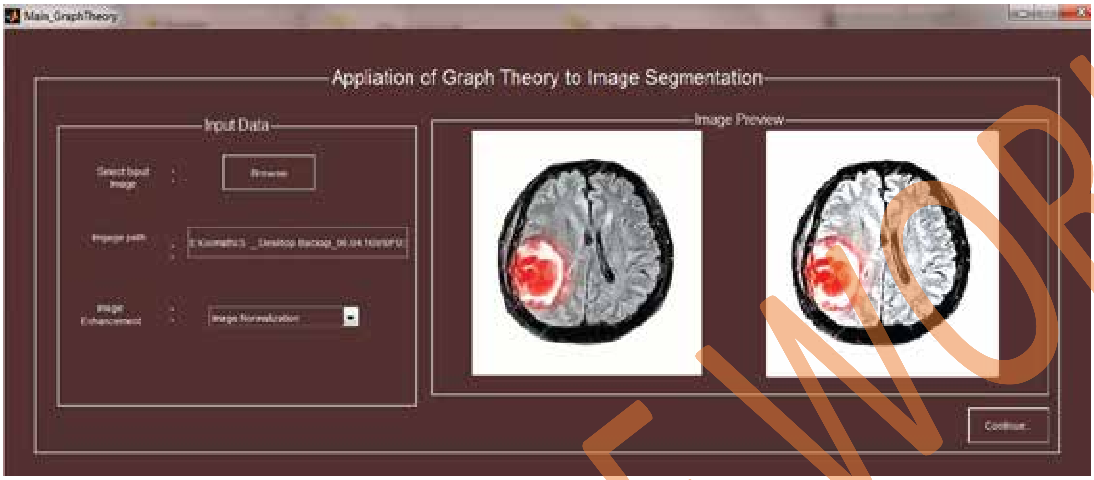
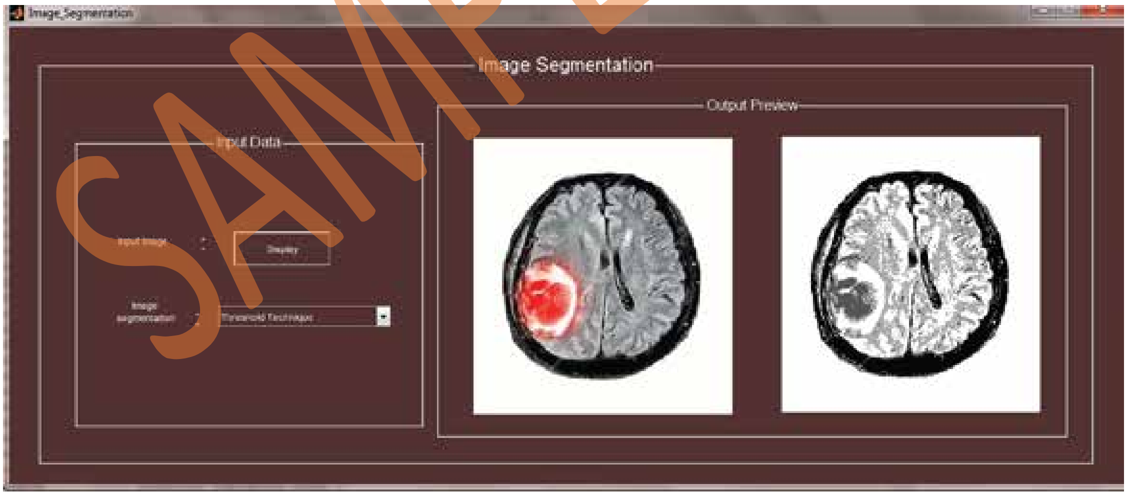


Figure 3: Thresholding



As discussed in the OSTU thresholding, need to select the single threshold value T by converting the gray level image into a binary image. Then the gray level values below T will be classified as black and above the value is white.

4.3.2 K-Means Segmentation

K-Means algorithm is an unsubstantiated clustering algorithm that classifies the input data points into multiple classes based on their genetic distance from each other. This algorithm assumes the data features that form a vector space and to find natural clustering (Tatiraju & Mehta, n.d.). The points is clustered nearby centroids μi i = 1. K that are found by minimizing the objective

$$V \quad {}^{k}_{i \;\; 1 x_i \;\; S_i} \quad (x_i \quad {}_i)^2$$

Where there are k clusters S_i , i = 1, 2, . . . , k and μi are the centroid or mean point of all the points x_j $S_{i.}$

For the k-means, segmentation computes the intensity distribution (also called the histogram) of the intensities and initialize the centroids with k random intensities. Cluster the points based on the distance of their intensities from the centroid intensities. The distance between the each pixel pair is evaluated by means of varying intensity level of the pixels.

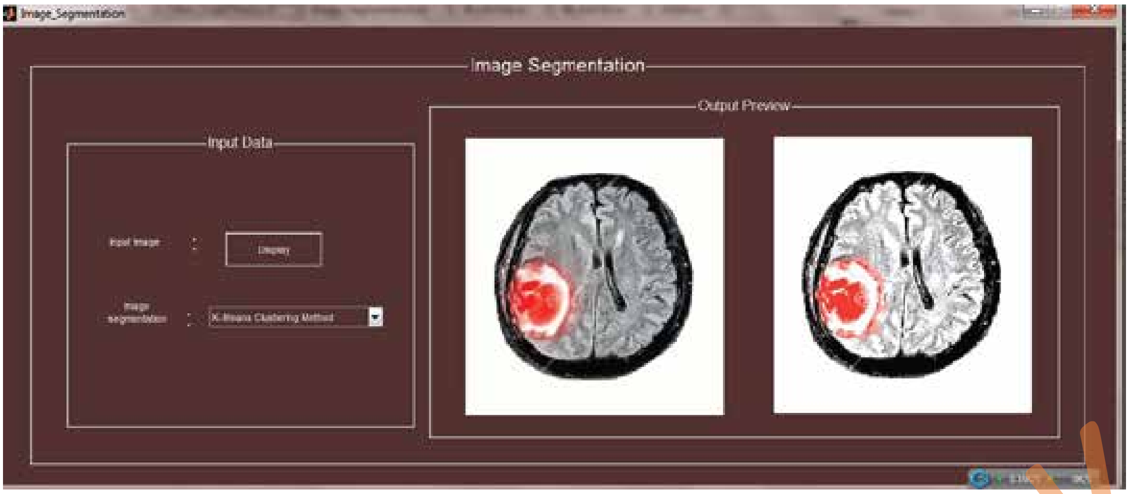
$$c^{(i)} : \quad \arg\min \left\| x^{(i)} \quad {}_j \right\|^2$$

Compute the new centroid for each of the clusters.

$${}_i : \quad \frac{{}^{m}_{i}\,{}_1 1 \; c_{(i)} \quad j \;\; x^{(i)}}{{}^{m}_{i}\,{}_1 1 \; c_{(i)} \quad j}$$

Where k is a parameter of the algorithm (the number of clusters to be found), i iterates over the all the intensities, j iterates over all the centroids and μ_i are the centroid intensities. The simulation result of k-means clustering shown in figure 4.

Figure 4: K-means Clustering

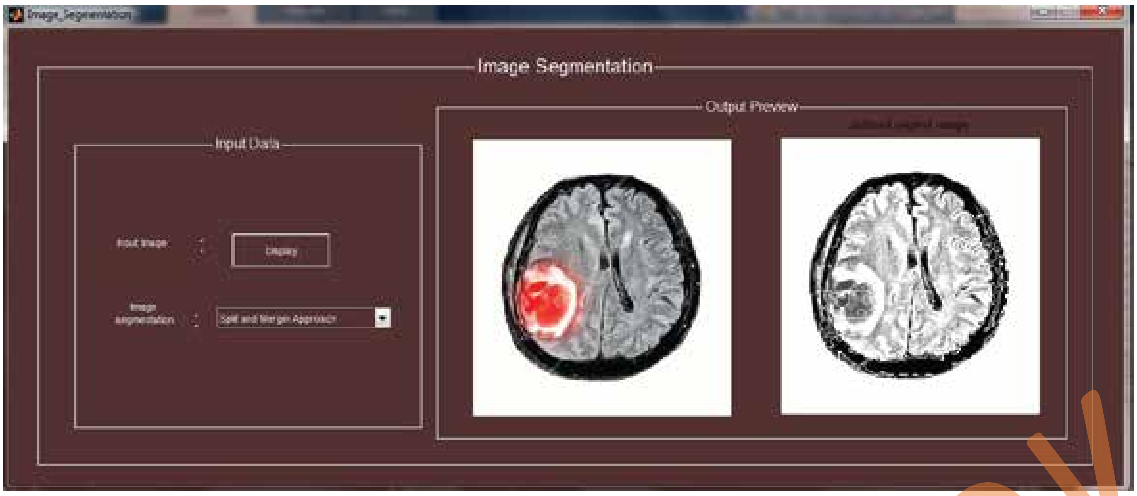


The segmented results are shown above in the figure. Here the K cluster centers are picked randomly, and need to allocate each pixel of the image to the cluster that reduces the distance between the pixel and the cluster center. Finally, calculate the cluster centers again by averaging all of the pixels in the cluster.

4.3.3 Split and Merging Technique

This image segmentation method is based on a quadtree barrier of an image. Therefore, it is sometimes called quadtree segmentation method. In this method, an image is represented as a tree, which is a connected graph with a number of cycles (Chaudhuri & Agrawal, 2010). The technique begins at the root of the tree. If it starts with non-uniform, the split & merge algorithm have two phases; the split and the merge. In the split phase, recursively split regions into four subregions in anticipation of our homogeneity criterion is met in all subregions. Conversely, if four son-squares are identical (homogeneous), then they can be merged as some connected components. This process is called as the merging process. The segmented region is the node of a tree. This process (splitting and merging) is continued recursively so that no further splits or merges are possible. The simulation result of split and merge method shown in figure 5.

Figure 5 : Split and merge approach



The pixels of the region are split based on their distance to the nearest height point and the plane to which that particular height point belongs. A new label is given to the pixels that belong to the new region. The new regions are stored as the new classes in the data structure, and their fields are updated. The merge process searches for neighboring regions whose associated points fall on the same plane. These are undergrown regions, and a co-planarity check determines whether they can be merged.

4.3.4 Normalized Cuts Method for Image Segmentation

In this method, the image is treated as a graph partitioning problem and offers a novel global method, the normalized cut, for segmenting the graph into regions/ segments. This technique not only measures the total dissimilarity between different regions and similarity within the regions. This method can be used in many applications such as segmenting immobile images and motion sequences.

Let us consider a graph G= (V, E) is partitioned into two disjoint sets, A, B, A B=V, A B by simply removing edges connecting the two parts. The degree of dissimilarity between these two pieces can be computed as the total weight of the edges that have been removed. In graph theory, it is known as a cut.

$$(\quad) \quad \Sigma \quad (\quad)$$

The normalized cut segmentation is defined as the value of total edge weight connecting the two partitions; the measure computes the cut cost as a fraction of the total edge connections to all the nodes in the graph. This disassociation measures the normalizedcut and it is mathematically written as (Shi & Malik, 2000; Jianbo Shi & Malik, 2000)

$$(\quad) \quad \frac{(\quad)}{(\quad)} \quad \frac{(\quad)}{(\quad)}$$

The normalized cut segmentation should possess two points, the first one is, a cut penalizes large segments, and another one is, fix by normalizing for the size of segments.

$$(\quad) \quad \frac{(\quad)}{(\;)} \quad \frac{(\quad)}{(\;)}$$

Where, volume (A) = sum of costs of all edges that touch A,

Volume (B) = sum of costs of all edges that touch B

$$(\quad) \quad \Sigma \quad (\quad)$$

The minimization of Equation can be formulated as a generalized eigenvalue problem, which has been well-studied in the field of spectral graph theory. The output result of normalized cut segmentation has shown in figure 6.

NCUT Segmentation Algorithm

1. Set up problem as G = (V,E) and define affinity matrix A and degree matrix D
2. Solve (D – A)x = λDx for the eigenvectors with the smallest eigenvalues
3. Let x^2 = eigenvector with the 2 nd smallest eigenvalue λ^2
4. Threshold x^2 to obtain the binary-valued vector x'^2 such that ncut(x'^2) ≥ ncut(x t ^{2}) for all possible thresholds t
5. For each of the two new regions, if ncut< threshold T, then recourse on the region

Figure 6: Normalized cut segmentation

In the experimental results shows, this method successfully separates the pixel from the image, which is connected to the pixel with similar intensity. The each pixel as a node and connecting each pair of pixels by an edge. The weight on that edge should reflect the likelihood that the two pixels belong to one object.

4.3.5 Efficient Graph-Based Image Segmentation

In the graph-based approach, a segmentation S is a partition of V into components, and its corresponding each component C S corresponds to a connected component in a graph G′ = (V, E ′), where E′ E. In graph based segmentation it is induced by a subset of the edges in E to measure the quality of elements in different ways with general components dissimilarities. This means that edges between two vertices in the same component should have relatively low weights, and edges between vertices in different components should have higher weights.

The pairwise predicate for segmentation is (Felzenszwalb & Huttenlocher, 2004; Narang & Rathinavel, n.d.),

$$D(C_1 \quad C_2)\{\underset{false}{true}\, if\underset{otherwise}{Dif(C_1,C_2)} \quad \min t(C_1,C_2)$$

Where (C1 , C2) - the difference between two components.

Mint (C1 , C2) is the internal different in the components C1 and C2

The region comparison predicate evaluates if there is evidence for a boundary between a pair or components by checking if the difference between the components, Dif(C_1, C_2), is large relative to the internal difference within at least one of the components, Int(C_1) and Int(C_2).

$$Dif(C_1,C_2) \quad \min_{v_i \; C_1, V_j \; C_2, (V_i,V_j) \; E} (V_i,V_j)$$

The different between two components is the minimum weight edge that connects a node vi in component C1 to node vj in C2

$$Int(C) \quad \max_{e \; MST(C,E)} (e)$$

Here Int(C) is to the maximum weight edge that connects two nodes in the same component.

This method Capture perceptually important Groupings and is highly efficient. The simulation results are shown in figure 27. The implementation steps are discussed as follows.

Algorithm Implementation Steps:

The input is a graph G = (V, E), with n vertices and m edges. The output image is a segmentation of V into components $S = (C_1, \ldots, C_r)$.

0. Sort E into $\pi = (o_1, \ldots, o_m)$, by non-decreasing edge weight.

1. Start with a segmentation S^0, where each vertex v_i is in its own component.

2. Repeat step 3 for q = 1, . . . , m.

3. Construct S_q using S_{q-1}

4. Return $S = S_m$.

A segmentation S is a partition of V into components such that each component (or region) C S corresponds to a connected component in a graph G′ = (V, E′), where E′ E.

Goal is to have the elements in one component to be similar, and elements in different components to be dissimilar

4.3.6 The Iterated Graph Cuts Method

The iterative graph cut algorithm starts from the sub-graph that comprises the user labeled foreground/background regions and works iteratively to label the nearby un-segmented regions. In each iteration, only the local neighboring regions to the labeled regions are complicated in the optimization so that much interference from the far unknown regions can be significantly reduced (Peng et al., 2010; Nagahashi et al., 2007; Chang & Chou, 2014).

Let use consider the smallest possible sub-graph is selected, and graph-cut is run, and the residual graph is obtained. Then, solution for a subset of connected nodes R having the same segmentation result cannot be changed simultaneously by the external flow. These changes correspond to the flipping the label of all nodes in region R.

If F is foreground,

$$_{i \quad R} \quad ^{iS} \quad ^{iT} \quad _{iR,j \quad R} \quad ^{ij}$$

If R is background,

$$_{i \quad R} \quad ^{iT} \quad ^{iS} \quad _{j \quad R,i \quad R} \quad ^{ji}$$

w_iS and w_iT denote the terminal weight of node i.

This condition holds since the cost of the changing the solution is larger than the cost of cutting all the non-terminal edges. Solution to the part of the nodes in R might still change; however, the result of the whole R cannot change.

Algorithm

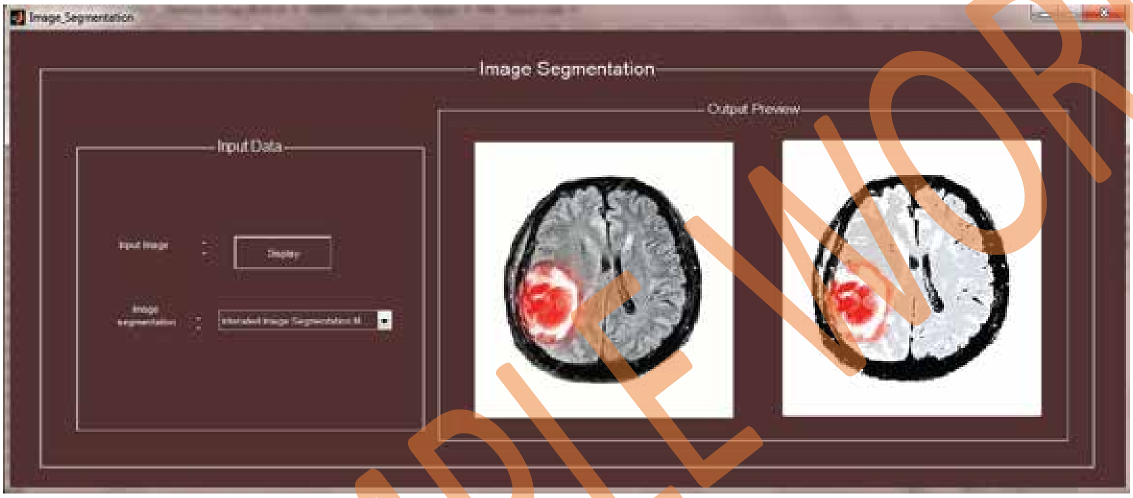
The input is mean shift initial segmentation of the given image and a graph G whose nodes consist of the user input foreground/background seed regions R.

The output is the segmentation result.

1. Add neighboring regions of foreground regions into G.

2. Construct foreground and background data models from seed regions R.
3. Use graph cuts algorithm to solve argmin f
4. Add background and foreground regions resulting from step 3 into R.
5. Add adjacent regions of the foreground seeds into G.
6. Go back to step 2 until no adjacent regions can be found.
7. Set labels of the remaining regions

Figure 7: Iterated Graph cut



The experimental results show the black nodes represent the ground-truth foreground and grey nodes represent the ground-truth background nodes.

4.3.7 Segmentation Using Minimal Spanning Trees

A graph G = (V, E), a spanning tree is a tree that spans all the nodes. In other words, it is a tree on all the nodes V. Evert connected graph has a spanning tree. Some graphs may have several spanning trees. In fact, you already saw Cayley's formula, which says that the complete graph K_n on n vertices has n^{n-2} spanning trees.

Formally, for a graph G = (V, E), the spanning tree is E ′ E such that:

u V: (u, v) E′ (v, u) E′ v V, In other words, the subset of edges spans all vertices

Need to find the spanning tree with the least cost, where the cost of the spanning tree T = (V, E′) is $e \quad E'$, c_e, the sum of its edge costs. Frequently the minimum-cost spanning tree gets shortened to “minimum spanning tree” or “MST.

In order to addresses, the problem of segmenting Kruskal's algorithm is applied to an image into regions by defining a predicate for measuring the evidence for a boundary between two regions using a graph-based representation of the image. An important characteristic of the method is its ability to preserve detail in low-variability image region while ignoring detail in the high-variability image region. This algorithm finds a minimum spanning tree for a connected weighted graph. This means it finds a subset of the edges that forms a tree that includes every vertex, where the total weight of all the edges in the tree is minimized. If the graph is not connected, then it finds a minimum spanning forest (a minimum spanning tree for each connected component) (Peter, 2010).

The image to be segmented is subjected to background elimination and then represented as an undirected weighted graph G. Here each pixel is measured at one vertex of the graph and the edges are drawn based on the 8-connectivity of the pixels. The weights are assigned to the edges by using the absolute intensity difference between the adjacent pixels. The segmentation is achieved by effectively generating the Minimal Spanning Tree and adding the non-spanning tree edges of the graph with selected threshold weights to form cycles sustaining the certain criterion, and each cycle is treated as a region. The neighboring cycles recursively merge until the stopping condition reaches and obtains the optimal region based segments. This proposed method is able to locate almost proper region boundaries of clusters and is applicable to any image domain (Janakiraman & Mouli, 2007).

Algorithm for finding MST using Kruskal's method

1. Sort all the edges in an image as the non-decreasing order of weight.
2. Pick the smallest edge. Check if it forms a cycle with the spanning tree formed so far. If the cycle is not formed, include this edge. Else, discard it.
3. Repeat step#2 until there are (V-1) edges in the spanning tree.

Figure 8: Minimal Spanning Tree

The image to be segmented is subjected to background elimination and then represented as an undirected weighted graph G. Here each pixel is measured at one vertex of the graph and the edges are drawn based on the 8-connectivity of the pixels. The weights are assigned to the edges by using the absolute intensity difference between the adjacent pixels. The segmentation is achieved by effectively generating the Minimal Spanning Tree and adding the non-spanning tree edges of the graph with selected threshold weights to form cycles sustaining the certain criterion, and each cycle is treated as a region. The neighboring cycles recursively merge until the stopping condition reaches and obtains the optimal region based segments. This proposed method is able to locate almost proper region boundaries of clusters and is applicable to any image domain.

4.3.8 Image Segmentation Using Euler Graphs

A closed walk in a graph G containing all the edges of G is called a Euler line in G. A graph containing a Euler line is called an Euler graph. For instance, know that a walk is always connected. Since the Euler line (which is a walk) contains all the edges of the graph, a Euler graph is connected except for any isolated vertices the graph may contain. As isolated vertices do not contribute anything to the understanding of a Euler graph, it is assumed now onwards that Euler graphs do not have any isolated vertices and are thus connected (Zhu et al., 2013b; Janakiraman & Chandra Mouli, 2010; Agarwal & Singh, 2009).

For example that a graph has a Euler path P with vertex v other than the starting and ending vertices, the path P enters v the same number of times that it leaves v. Therefore, there are 2s edges having v as an endpoint.

This algorithm for image segmentation problem uses the concepts of Euler graphs in graph theory. By treating the image as an undirected weighted non-planar finite graph (G), image segmentation is handled as graph partitioning problem. This method discovers region boundaries or clusters and runs in polynomial time. Subjective comparison and objective evaluation show the efficiency of the proposed approach in different image domains. As per this process, the segmented result are shown in figure 9.

Algorithm

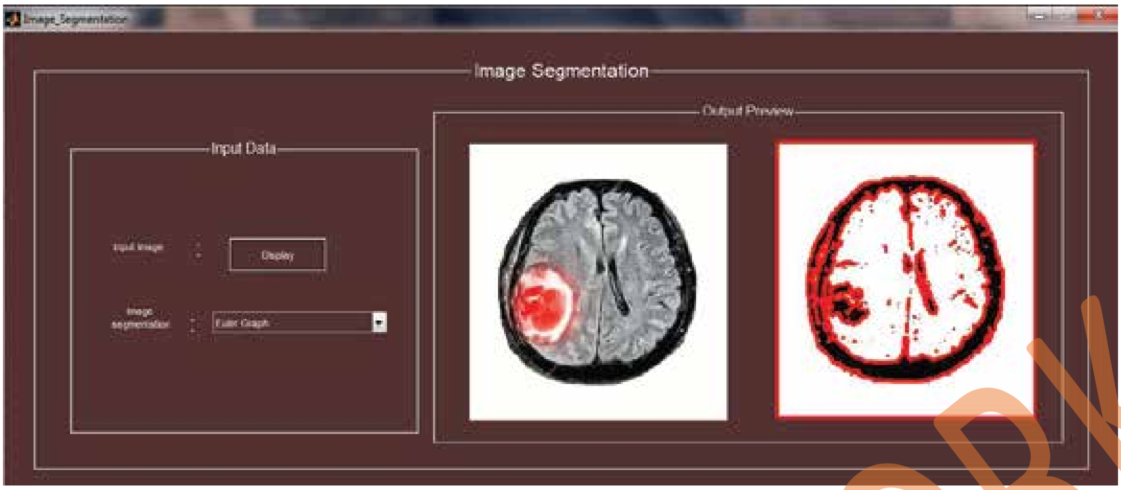
Step-1: Representation of image as a grid graph

Step-2: Conversion of grid graph into Eulerian

Step-3: Segmentation Procedure

Step-4: Refinement of segments

Figure 9: Euler Graph



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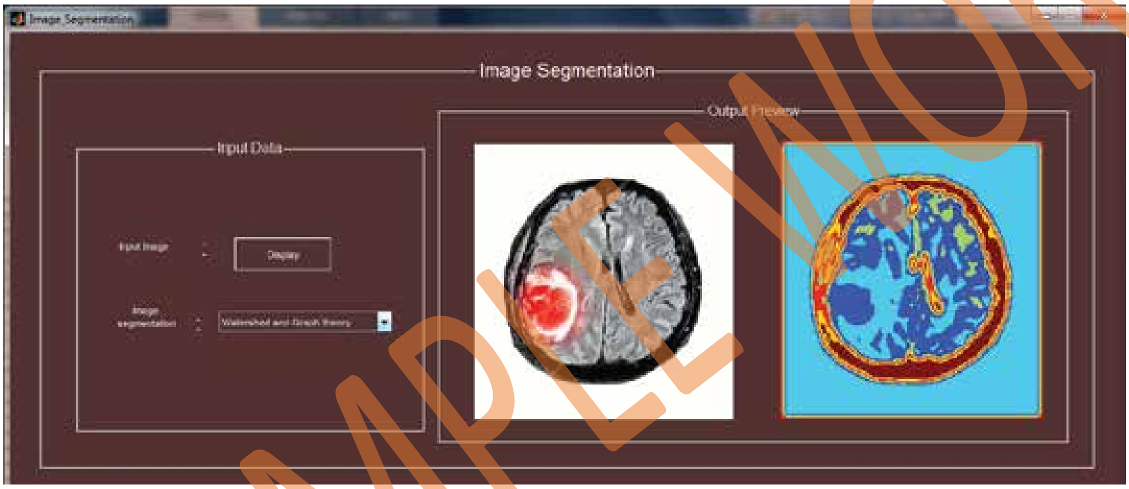
4.3.9 Segmentation Method Based On Grey Graph Cut

The image segmentation method based on gray graph cut is used for improving the performance of image segmentation, which integrates gray theory and graphs cut theory. Here the image is taken as a weighted undirected graph after that the relationships of grey-levels and positions in local regions are discussed via gray relational analysis, a gray weight matrix is established, based on which a gray partition function is constructed (Ma et al., 2010). Next, the image is binarized with the gray-level that corresponds to the minimum value of the gray partition function.

4.3.10 Combination of Watershed and Graph Theory

The extracting object of interest in medical images is challenging since strong noise, poor gray-scale contrast, blurred margins of tissue are characteristics of medical images. A segmentation approach that syndicates watershed algorithm with graph theory is proposed in this paper. This algorithm reconstructs gradient before watershed segmentation, based on the reconstruction, a floating-point active image is introduced as the reference image of the watershed transform.

Figure 10: Watershed segmentation



The extracting object of interest in medical images is challenging since strong noise, poor gray-scale contrast, blurred margins of tissue are characteristics of medical images. A segmentation approach that syndicates watershed algorithm with graph theory is proposed in this paper. This algorithm reconstructs gradient before watershed segmentation, based on the reconstruction, a floating-point active image is introduced as the reference image of the watershed transform. Finally, a graph theory-based algorithm Grab Cut is used for fine segmentation. False contours of over-segmentation are effectively excluded and total segmentation quality significant improved as suitable for medical image segmentation. The output result of proposed approach shown in figure 13. Subsequently, the pixels are segregated based on the color, shape and texture. While compared to existing approach, the proposed approach provide the effective results.

4.3.11 The Combination of Iterated Region Merging and Localized Graph Cuts Segmentation

The Peng *et al*. (2010) Graph cuts technique provides a globally optimal solution to image segmentation; however, the complex content of an image makes it hard to precisely segment the whole image all at once. The iterated conditional mode (ICM) proposed by Besag (1993) is a deterministic algorithm which maximizes local conditional probabilities sequentially. It uses the "greedy" strategy in the iterative local maximization to approximate the maximal joint probability of a Markov Random Field (MRF). Inspired by ICM, consider the graph cuts algorithm in a "divide and conquer" style: finding the minima on the subgraph and extending the sub-graph successively until reaching the whole graph. The proposed method works iteratively, in place of the previous one-shot graph cuts algorithm (Boykov & Jolly, 2001).

The proposed iterated region merging method starts from the initially segmented image by the modified watershed algorithm.In each iteration, new regions which are in the neighborhood of newly labeled object and background regions are added into the sub-graph, while the other regions keep their labels unchanged. The inputs consist of the initial segmentation from watershed segmentation and user marked seeds. The object and background data models are updated based on the labeled regions from the previous iteration.

Furthermore, evaluated the segmentation performance of the proposed method in comparison with the graph cuts algorithm (Boykov & Jolly, 2001) and GrabCut (Rother *et al*., 2004). Since they used watershed for initial segmentation, for a fair comparison, also extend the standard graph cuts to a region based scheme, i.e. use the regions segmented by watershed, instead of the pixels, as the nodes in the graph. GrabCut algorithm is also an interactive segmentation technique based on graph cuts and has the advantage of reducing user's interaction under complex background. It allows the user to drag a rectangle around the desired object. Then the color models of the object and background are constructed according to this rectangle. Hence, in total, have four algorithms in the experiments: the pixel based graph cuts (denoted by GCp), the region based graph cuts (GCr), the GrabCut and the proposed iterated region merging method with localized graph cuts (Peng et al., 2011).

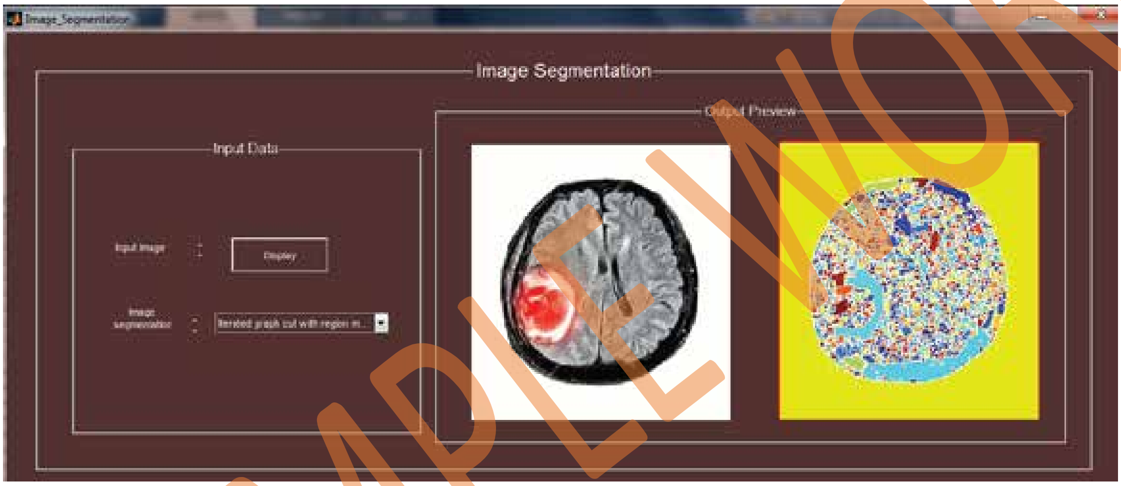
The iterated region merging based graph cuts algorithm which is an extension of the standard graph cuts algorithm. Graph cuts address segmentation in an optimization framework and finds a globally optimal solution to a wide class of energy functions. However, the extraction of objects in a complex background often requires a lot of user interaction. The algorithm starts from the user labeled sub-graph and works iteratively to label the nearby un-segmented regions. In each iteration, only the confined neighboring regions to the labeled regions are complicated in the optimization so that much interference from the far unknown regions can be significantly reduced. Meanwhile, the data models of the object and background are updated iteratively based on high confident labeled regions. The sub-graph requires less user guidance for segmentation and thus better results can be obtained with the same amount of user interaction. Experiments on benchmark datasets validated that this method yields much better segmentation results than the standard graph cuts and the GrabCut methods in either qualitative or quantitative evaluation.

Iterated region merging with localized graph cuts Algorithm1:

1. Build object and background data models based on labeled regions Ro and Rb.
2. Build subgraph G =< V , E >, where V consist of Ro,Rb, and their adjacent regions.

3. Update object and background data models using the SelectLabels() algorithm
4. Use graph cuts algorithm to solve the min-cut optimization on G , i.e. arg min f U(f|d, f k N).
5. Update object regions Ro and background regions Rb according to the labeling results from step 4.
6. Go back to step 2, until no adjacent regions of Ro and Rb can be found.
7. Return the segmentation results.

Figure 11: Iterated Graph cut with region merging



4.3.12 The Combination of Fuzzy and Graph-Theoretical Clustering Segmentation

Clustering is a process for classifying objects or patterns in such a way that samples of the same group are more similar to one another than samples belonging to different groups. Many clustering strategies have been used, such as the hard clustering scheme and the fuzzy clustering scheme, each of which has its own special characteristics (Yong, 2009). The conventional hard clustering method restricts each point of the dataset to exclusively just one cluster. As a consequence, with this approach the segmentation results are often very crisp, i.e., each pixel of the image belongs to exactly just one class. However, in many real situations, for images, issues such as limited spatial resolution, poor contrast, overlapping intensities, noise and intensity inhomogeneities variation make this hard segmentation a difficult task. Thanks to the fuzzy set theory (Zadeh, 1965) was proposed, which produced the idea of partial membership of belonging described by a membership function; fuzzy clustering as a soft segmentation method has been widely studied and successfully applied in image segmentation (Kwon *et al*., 2003; Tolias *et al*., 1998; Tolias & Panas, 1998; Noordam *et al*., 2000; Ahmed *et al*., 2002; Zhang & Chen, 2003; Li *et al*., 2003; Pham & Prince, 1999). Among the fuzzy clustering methods, the fuzzy c-means (FCM) algorithm (Bezdek, 1981) is the most popular method used in image segmentation because it has robust characteristics for ambiguity and can retain much more information than hard segmentation methods (Bezdek *et al*., 1993). Although the conventional FCM algorithm works well on most noise-free images.

Graph-theoretic definition of a cluster

The data to be clustered as an undirected edge-weighted graph with no self-loops G = (V,E,w), where V = {1,...,n} is the vertex set, E V × V is the edge set, and w : E IR + is the (positive) weight function. Vertices in G correspond to data points; edges represent neighborhood relationships, and edge-weights reflect similarity between pairs of linked vertices (Pavan & Pelillo, 2003). As customary, represent the graph G with the corresponding weighted adjacency (or similarity) matrix, which is the n × n symmetric matrix A = (aij) defined as:

$$aij \quad {}^{w(i,j),}_{0,} \; if\,(i,j) \quad E \text{ otherwise.}$$

Graph theoretic technique for metric modification such that it gives a much more global notion of similarity between data points as compared to other clustering methods such as k-means. It thus represents data in such a way that it is easier to find meaningful clusters on this new representation. It is especially useful in complex datasets where traditional clustering methods would fail to find groupings (Trivedi, 2012).

In the process of segmentation using traditional graph-theoretical clustering method is sensitive to noise and fuzzy edges. Thus false segmentation result appears. Further, the large computational complexity also affects its application.

FCM Algorithm:

Step 1: Set the cluster centroids vi according to the histogram of the image, fuzzification parameter q ($1 \leq q < \infty$), the values of c and $\varepsilon > 0$.

Step 2: Compute the histogram.

Step 3: Compute the membership function

Step 4: Compute the cluster centroids

Step 5: Go to step 3 and repeat until convergence.

Step 6: Compute the a priori probability with the obtained results of membership function and centroids.

Step 7: Recomputed the membership function and cluster centroids with the probabilities.

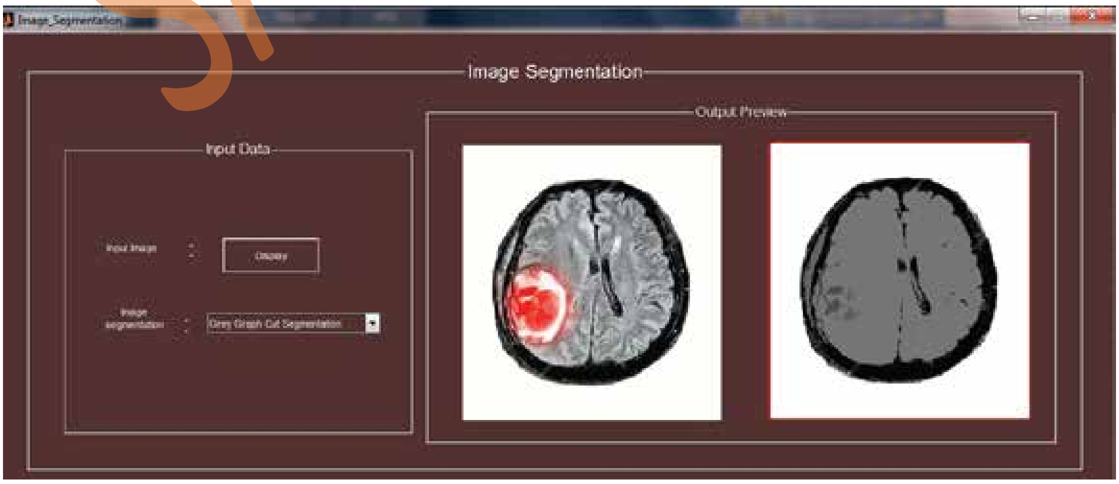
Step 8: If the algorithm is convergent, go to step 9; otherwise, go to step 6.

Step 9: Image segmentation after defuzzification and then a region labeling procedure is performed.

Graph-Theoretic:

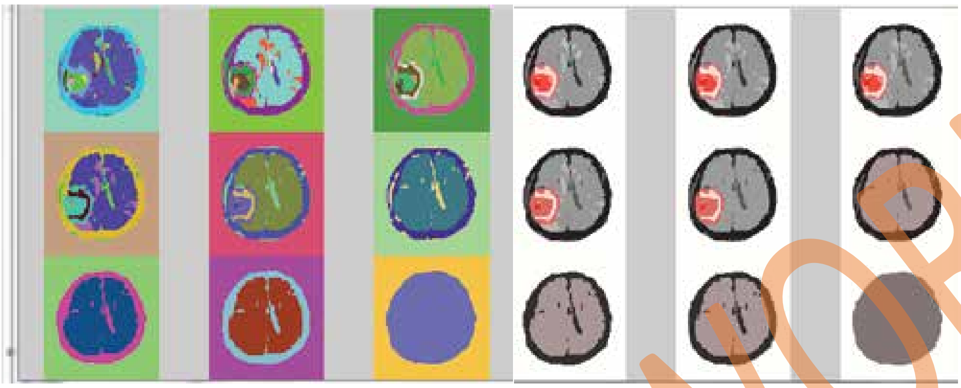
1. For the dataset having n data points, construct the similarity graph G. The similarity graph can be constructed in two ways: by connecting each data point to the other n – 1 data points or by connecting each data point to its k-nearest neighbors. A rough estimate of a good value of the number of nearest neighbors is log (n). The similarity between the points is matrix W.
2. Given the similarity graph, construct the degree matrix D.
3. Using D and W find Lsym.
4. Let K be the number of clusters to be found. Compute the first K eigenvectors of Lsym. Sort the eigenvectors according to their eigenvalues.
5. If u1, u2...uK are the top eigenvectors of Lsym, then construct a matrix U such that U = {u1, u2 . . . uK}. Normalize rows of matrix U to be of unit length.
6. Treat the rows in the normalized matrix U as points in a K-dimensional space and use k-means to cluster these.
7. If c1, c2 . . . cK are the K clusters, Then assign a point in the original dataset si to cluster cK if and only if the i th row of the normalized U is assigned to cluster cK.

Figure 12: cluster based segmentation



Experimental results on the visible light image and SAR image indicate this method, being superior to some existing methods like Otsu and Normalized Cut, etc., not only can segment the images with obvious difference between targets and backgrounds, but also suppress image noise effectively.

Figure 13: color spatial clustering with consensus region merging



(a) (b)

(c)

Furthermore, verified the proposed approach using normal x-ray images of lungs and ultrasound images. The performance of these images is discussed in table 2 and table 3. In the following section have discussed the image quality analysis approach with result of the same.

4.5 Image Quality analysis

Basically, quality assessment algorithms are needed for mainly three types of applications. These are discussed as follows:

For comparative analysis between different alternatives.

For optimization purpose, where one maximize quality at a given cost.

For quality monitoring in real-time applications.

Image Quality is a characteristic of an image that measures the perceived image degradation (typically, compared to an ideal or perfect image). Imaging systems may introduce some amounts of distortion or artifacts in the signal, so the quality assessment is an important problem. Image Quality assessment methods can be broadly classified into two categories. These are,

1. Objective measurement
2. Subjective measurement

Table 1: Various Quality Metrics for MRI Image

Quality Measure / Segmentation methods	MSE	PSNR	NCC	AD	SC	MD	NAE	LMSE	RMSE	SSIM
Threshold Technique	47.84	1.332	0.0159	203.2	39.1	251	0.9842	-0.75	218.74	0.0292
K-means clustering approach	49.02	1.227	0.003	205.75	65.11	254.0	0.996	-0.15	221.4	0.016
Split and merging method	9.10	18.53	1.004	-4.4416	0.9733	0	0.0215	0.1275	30.1729	0.9044
Normalized	39.5				1.0883	238	0.18			

cut segmentation	6	12.1583	0.9194	0.1117			99	0.1798	62.8964	0.5125
Iterated Graph cut segmentation	49.01	1.2278	0.0040	205.7426	6.2779e+04	254.0490	0.9960	3.5050e-04	221.3860	0.0167
Minimal spanning tree	4.9403e+04	1.1933	7.6938e-06	206.5464	2.3072e+06	255	1.0000	6.0663e-04	222.2679	0.0130
Euler Graph	4.9002e+04	1.2287	0.0041	205.7256	5.8661e+04	254	0.9959	6.4021e-04	221.3632	0.0166
Iterated Graph cut with region merging	7.8040e+03	9.2077	0.8862	-0.3459	1.0748	240	0.3118	0.2438	88.3400	0.4498
Grey Graph cut	3.2053e+03	13.0722	0.8673	29.3153	1.2509	241	0.1419	0.2141	56.6150	0.6982
Watershed with graph theory	2.7837e+04	3.6846	0.3531	105.8839	3.7097	212	0.7787	0.1730	166.8441	0.2098
Fuzzy with graph theory approach	2.6069e+03	13.9695	0.8111	39.6805	1.4816	137	0.2189	0.1292	51.0579	0.7246
color spatial clustering with consensus region merging	6.10	2.63	1.203	-2.4811	0.98933	0	0.1215	0.1037	26.1729	0.9844

4.6 Summary

In this research analyzed about the application and performance of the graph theory on image segmentation process is examined. This section provides the overview of methods adopted in this research for image segmentation technique. In order to provide a clear description of the process flow, this chapter is organized in a structured manner. Previously in this chapter described about the overall summary of graph theory and followed by its application to image segmentation processing. Also, this chapter provides the experimental analysis which means theoretical approach adopted(N-cut segmentation, k-means clustering) for image segmentation process in this research is explained. Finally, parameters considered for image segmentation performance evaluation also analyzed. The next chapter provides simulation measurement of the image segmentation process is presented.

In the previous research, most of them have focused towards graph based approach to image segmentation. From these, have identified the issues and drawbacks. This is discussed as follows. Few of them have focused graph based segmentation on a medical application such as Lürig *et al*. (1997), Chen *et al*. (2006), Elmasry *et al*. (2012), Kapade *et al*. (2014), Srinivasan *et al*. (2014). However, they considered only gray scale image (only on brain or lungs image) or color image with a specific application. In specifically they not address the most challenging case of segmenting retinal layers in human eyes with multiple types of pathology from different diseases such as diabetic retinopathy, macular hole, and age-related macular degeneration. Also, they fail to determine ranges of characteristic values for interesting medical features Lürig*et al*. (1997).On the other hand, few of them has focused towards segmentation based remote sensing application such as Mercovich *et al*. (2011), Dezső *et al*. (2012), Yang *et al*. (2015). However, this study fails to speed up the segmentation algorithm. Also, does not focus towards texture features to compute the heterogeneity. Liu *et al*. (2010), Pavan and Pelillo (2003), Elmasry *et al*.(2012),Yang *et al*. (2015)they recommended concentrating more towards pattern recognition domains such as texture segmentation, perceptual grouping, and the unsupervised organization of an image database. Also need to focus on region-based approaches. Peng *et al*. (2013) conducted a survey of graph theoretical methods for image segmentation. However, the study fails to provide versatile graph-based algorithms for a wide range of practical applications. Pham *et al*. (2014) method needed to improve especially for the extraction of representative pixels and the definition of vertex description vectors supporting the construction of graph. This proposed method does not carry the other types of VHR images. Mishra *et al*. (2014), Sarsoh *et al*. (2012) the method is sensitive to unusual cases when the background in consecutive frames changes rapidly in addition to the appearance and disappearance of multiple objects in the same scene and using multiple cameras. Kale *et al*. (2015) an application of isoperimetric algorithm of graph theory for image segmentation and analysis of different parameters used in the algorithm. However, it fails to define function classes for Prim's minimum spanning tree, depth-first search, Dijkstra's shortest path algorithm, and Kruskal's minimum spanning tree. In addition need to designs for the graph traits classes will be made more generic and user defined. This way the application of all the graph classes will be truly generic, and graph theory can be applied easily for image analysis.Dikholkar *et al*. (2015)Due to that NCUT method using image pixel for segmentation, there are exponential numbers of possible partitions of the graph. As a result, it is computationally expensive to find the optimal partition (Zhao, 2015). The limitation of the Graph cut optimization approach is that it requires the number of partitions to be provided by users and hence cannot fully automatically segment an image. How to automatically determine the proper partition number for different images will be studied in the future. In addition, will explore the possibility of incorporating the proposed recursive calculation scheme into other information entropy methods (Yin *et al*., 2014).

In this thesis have proposed color spatial clustering with consensus region merging for segmenting the image and segmentation results are illustrated in Fig 2-12. Furthermore, compared the results of our algorithm on various color images with results, which is run using its default parameter. Additional images (CT scan and ultrasound images) were also used in our comparison; however, due to space consideration results of those are not presented. In this image segmentation, a merging strategy is joined the most coherent adjacent regions together. The predicted structure represents the region partition of the image and Merging more adjacent regions will produce a less representative segmentation. The most similar pair of adjacent regions corresponds to the edge with the minimum cost. This cost associated with each edge is therefore very important in order to define which regions will be merged. A merging algorithm removes some of the links and merges the corresponding nodes. Subsequently, the pair of most similar regions is merged until a termination criterion is reached. Here, the merging order is based on a measure of similarity between adjacent regions. At each step the algorithm looks for the pair of most similar regions (the edge of minimum cost). Since at each merging step the edge with the minimum cost is required, the appropriate data structure to handle the edge weights is a queue. Each edge node is inserted in the queue at the position defined by the merging order. An interesting representation of the queue is a balanced binary tree which is very efficient for managing a high number of nodes with fast access, insertion, and deletion. The region termination criterion defines when the merging ends. Usually, this criterion is based on *a priori* knowledge: the number of regions and other subjective criteria involving thresholds. Ideally, for an automatic segmentation, the termination criterion should be based on image properties which define the fact that the segmentation obtained is considered as "good" The segmentation evaluation provides a value which decreases the better the segmentation is. Therefore merging regions gives a better representative segmentation and the evaluation criterion decreases. The edges of the queue are processed one by one until a stabilization of the segmentation evaluation criterion. In order to assess the influence of the different representations of color, have to compare the obtained segmentations in several color spaces. To see if segmentation is close to the original image, an error metric (Image quality analysis) is needed. The error between the original image and the quantized image (obtained by associating the main color of each region to each segmented pixel) was used. In this thesis have used various quality matrices like MAE, MSE, RMSE, LMSE, PSNR, SC, SD, AD, NAE and SSIM. The results are discussed in table 1, 2 and 3. From this analysis, the proposed has better efficiency compared to the existing segmentation methods. Experimental results indicate that MSE and PSNR are very simple, easy to implement and have low computational complexities. But these methods do not show good results. MSE and PSNR are acceptable for image similarity measure only when the images differ by simply increasing the distortion of a certain type. But they fail to capture image quality when they are used to measure across distortion types. SSIM has widely used method for measurement of image quality.

CHAPTER VI: CONCLUSION AND RECOMMENDATIONS

In this present research was briefly discussed and analyzed the conventional with graphical based image segmentation method using graph theory. The implementation of various image segmentation, using graph theory was done. Some of the natural images used for the segmentation which is taken from the biomedical Dataset. The image data were captured using a data collection platform based on computer vision. Images were selected arbitrarily for our pre-segment approach. Subsequently, implemented the segmentation algorithm in MATLAB programming language based on the max-flow/min-cut code. In the experiments, investigated the effect of performance on the following aspects: (1) energy function, (2) the comparison of the method with the conventional methods, and (3) the quantitative evaluations.

For the image segmentation experiments, used color images from the biomedical database by using Google search. It contains 79 color images for training and testing. For each image, resized the input image and generated data per image. In this experiment, first sampled each image pixels, resulting in the reshaped image of size. Then, applied graph theory to segment images. To build descriptors of pixels, we computed a local color histogram. The indexed image is converted from the RGB input image with the minimum variance color quantization of various levels. Furthermore, applied the graph theory to image segmentation based on brightness, color, texture, or motion information. In the monocular case, construct the graph G =V; E by taking each pixel as a node and define the edge weight wij between node i and j as the product of a feature similarity term and spatial proximity term. Through the analysis of both theoretical and practical implementation, concluded as the proposed hybrid approach which is a combination of color spatial clustering with consensus region merging for segmenting the image is an efficient method for solving the image segmentation problem. Also, possible to obtain a good classification as well as segmentation accuracy. Additionally, the proposed color spatial clustering with consensus region merging segmentation algorithm improves upon the two primary drawbacks of the existing segmentation approach. Firstly, the regions are modeled in RGB space, secondly, a natural intuitive parameter for achieving good segmentation for a wide variety of images. All these results have been obtained without sacrificing the computational efficiency. Furthermore, have validated this algorithm by testing on a wide variety of medical images. The performance of our algorithm is clearly superior to existing algorithm in most cases. The simulation results have shown that it can be applied for major medical analysis applications. And it can also be used for general calculations. The implementation has been made flexible in order to allow it to be applied to varying problems. Looking at the results it is easy to conclude that graph cuts are by far the best algorithm because it produces the best segmentation. However, this approach has a distinct advantage because it uses user input. Additionally, proposed method run significantly faster than the existing methods because the input to the graph cuts method includes seed pixels which considerably decrease the space of possible segmentations. Although, one of the main things to notice from the table is how fast the proposed method is in comparison to the others

6.1 Advantages

Validated the performance of various segmentation methods using different medical images like MRI, CT scan, and ultrasound images.

High efficiency compared to existing segmentation method

6.2 Recommendation for Future work

Following are some of the tasks that plan to undertake in the near future to improve the results of this thesis:

Run the segmentation algorithm on the new dataset and evaluate its performance.

To measure the effectiveness of proposed method based on the size of input image like 5KB, 5MB, 10 MB and so on.

To apply this approach towards some other domains like environmental protection, disaster monitoring and so on.

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